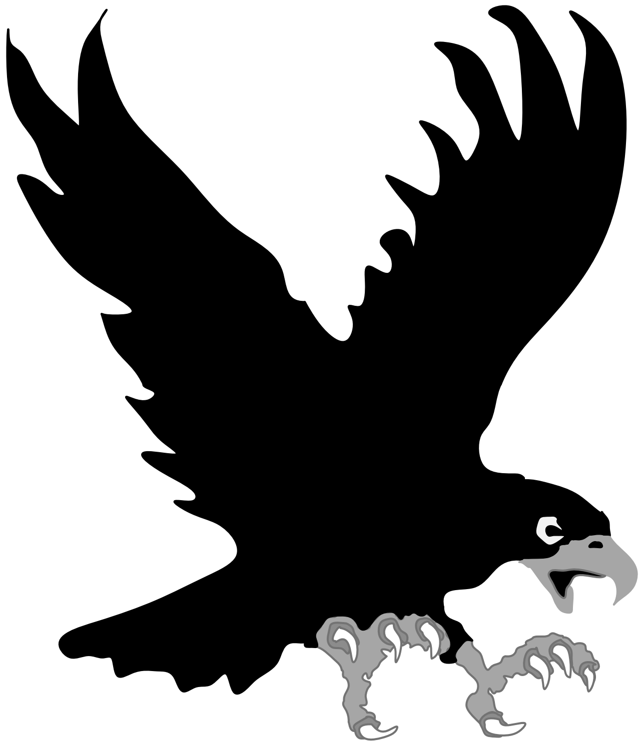


Programlama -1

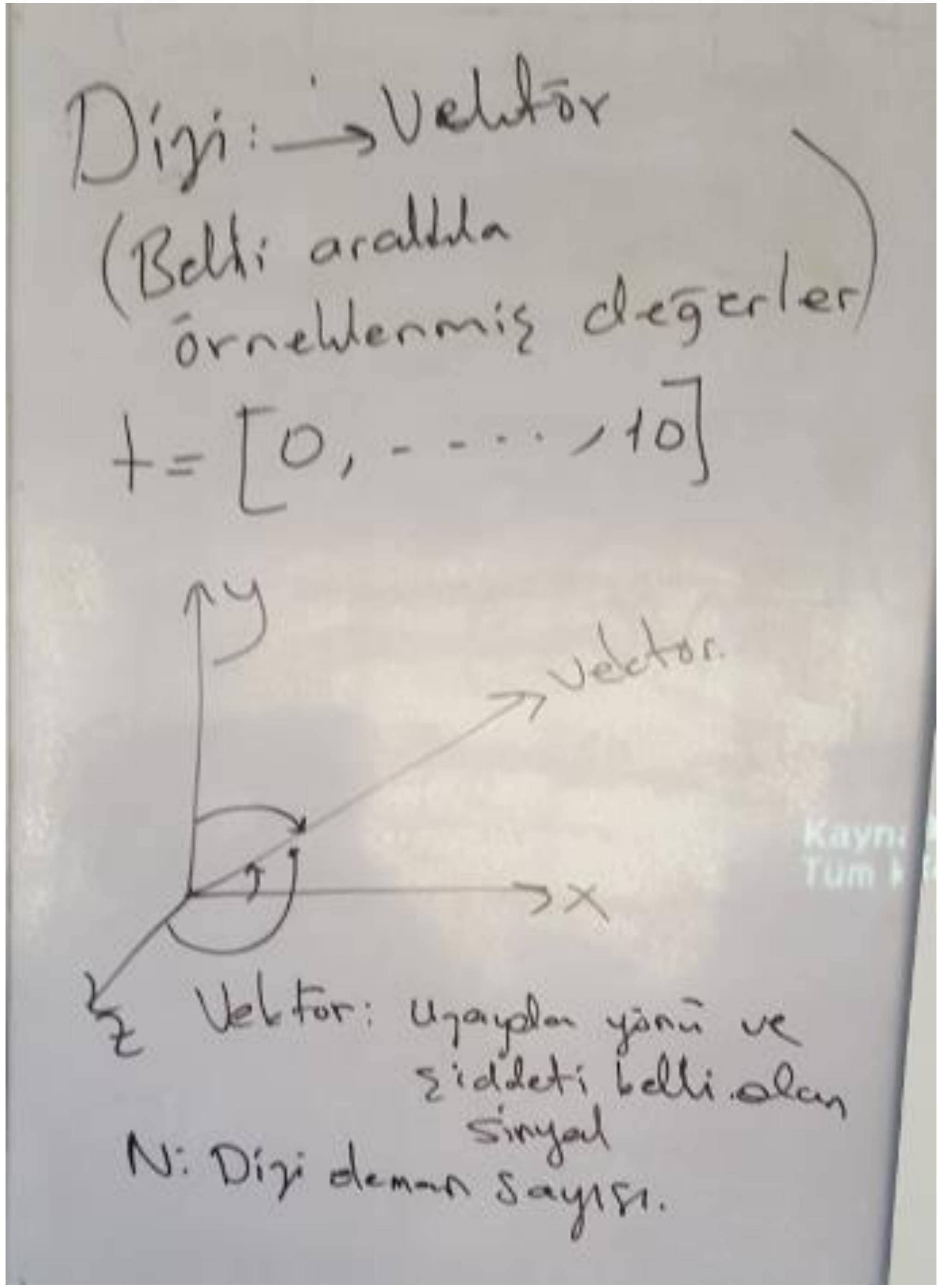
“Vector Analysis”

Dr. Cahit Karakuş, 2020

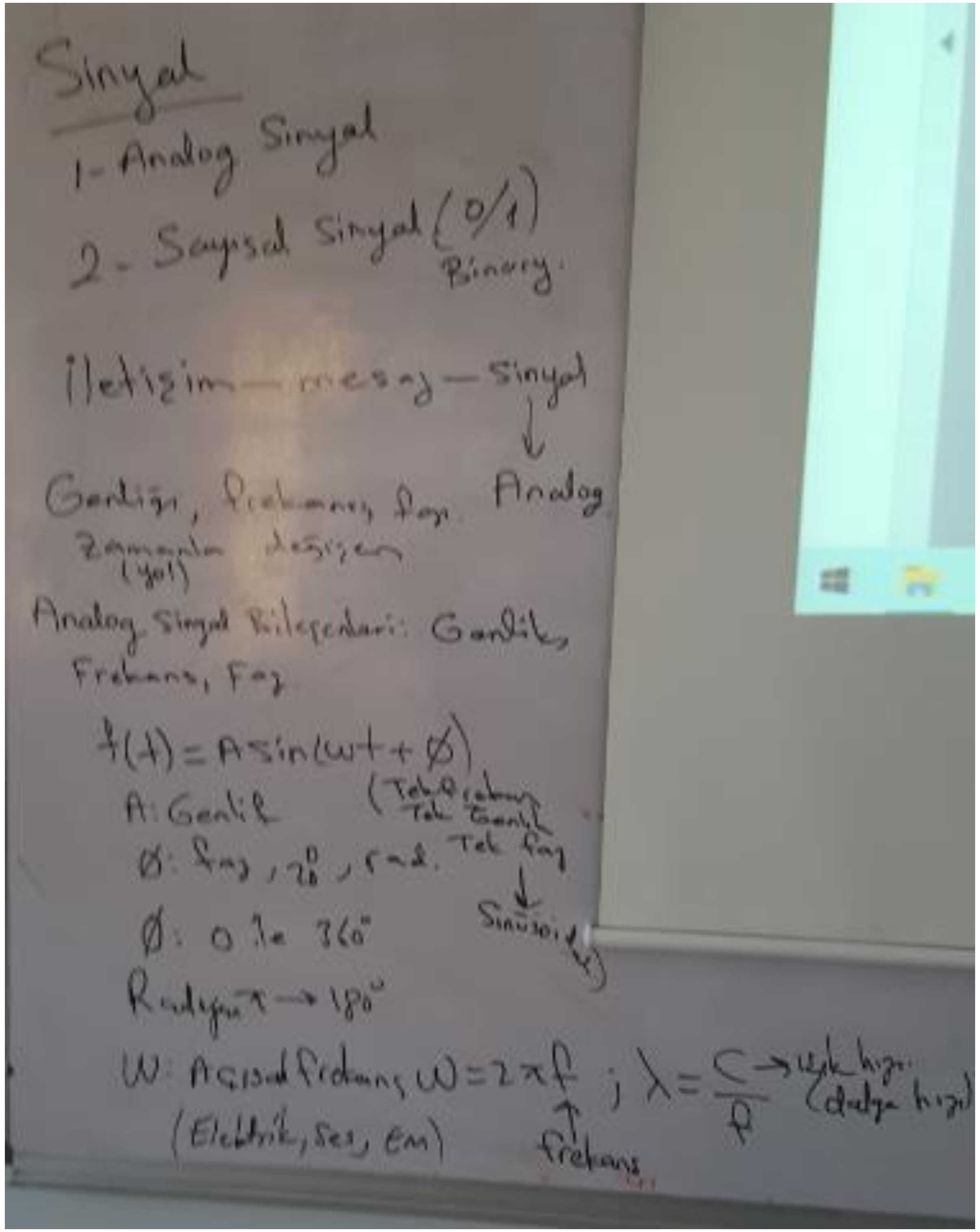
Vector Analysis

- Vektör terimi, matematik, mühendislik ve bilimin farklı alanlarında biraz farklı anlamlara sahiptir. Şimdiye kadar metin boyunca terim, matris cebirinin standart kurallarına göre satır veya sütun matrislerine atıfta bulunmak için kullanıldı ve bu kurallar da MATLAB tarafından kullanıldı.

- Vektör için yaygın olarak kullanılan bir diğer tanım, içinde yaşadığımız üç boyutlu koordinat sistemi açısından belirli yönleri olan uzamsal büyüklüklerle ilişkilendirilir.
- Bu miktarlara örnek olarak kuvvetler, hızlar, yer değiştirmeler, elektrik alanları, manyetik alanlar ve diğer birçok fiziksel değişken verilebilir.
- Üç boyutlu bir uzamsal vektör, MATLAB'da bir satır veya sütun vektörü cinsinden temsil edilebilir.
- Bu nicelikleri tanımlamada yararlı olan belirli matematiksel işlemler vardır ve konu alanı vektör analizi olarak adlandırılır.

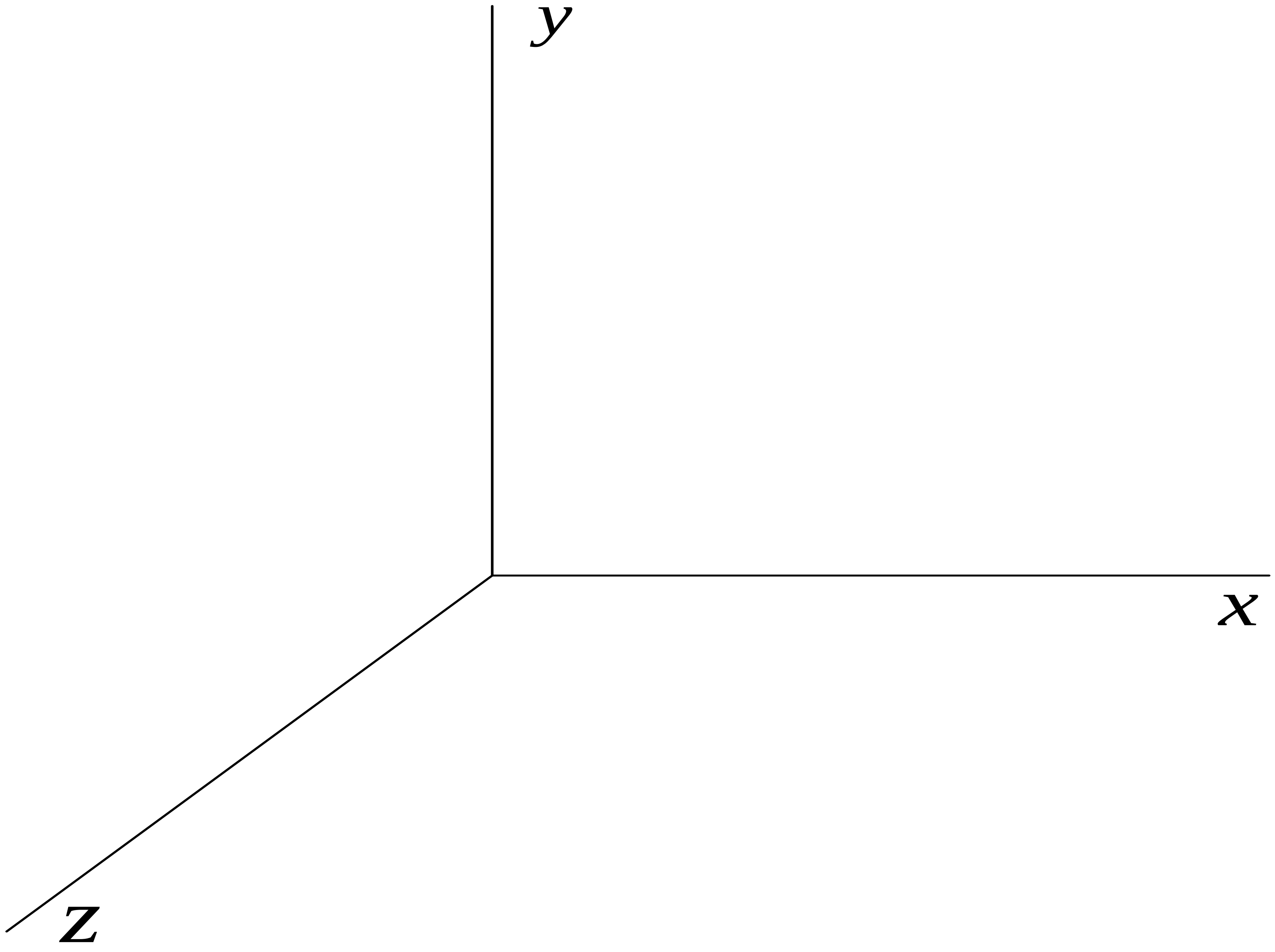


Dizi: → Vektör
(Belli aralıkta
örneklenmiş değerler)
t = [0, ... , 10]
y
vektör
x
z
Vektör: Uzayda yönü ve
şiddeti belli olan
sinyal
N: Dizi eleman sayısı.

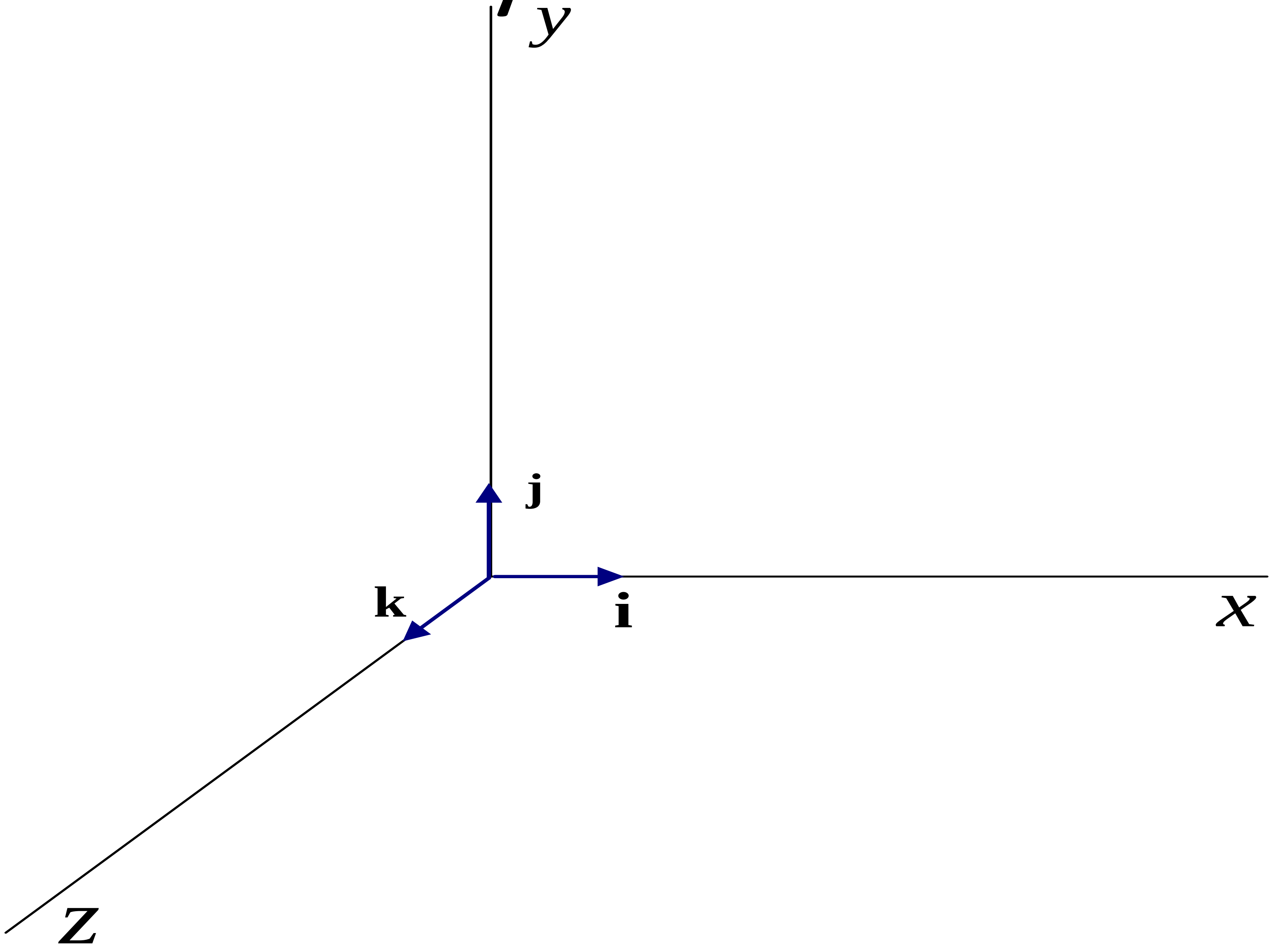


Sinyal
1- Analog Sinyal
2- Sayısal Sinyal (0/1)
Binary
İletişim — mesaj — sinyal
Analog
Genliği, frekansı, faz
Zamanla değişen
(yol)
Analog Sinyal Bileşenleri: Genlik,
Frekans, Faz
f(t) = A sin(wt + Ø)
A: Genlik
(Tek frekans
Tek Genlik
Tek faz
Sinüsoidal)
Ø: faz, 2°, rad.
Ø: 0 ile 360°
Radyan π → 180°
W: Açısal frekans W = 2πf ; λ = c / f
frekans
(Elektrik, ses, em)
ışık hızı
(dalga hızı)

Rectangular Coordinate System



Unit Vectors in Rectangular Coordinate System



Vector Representation:

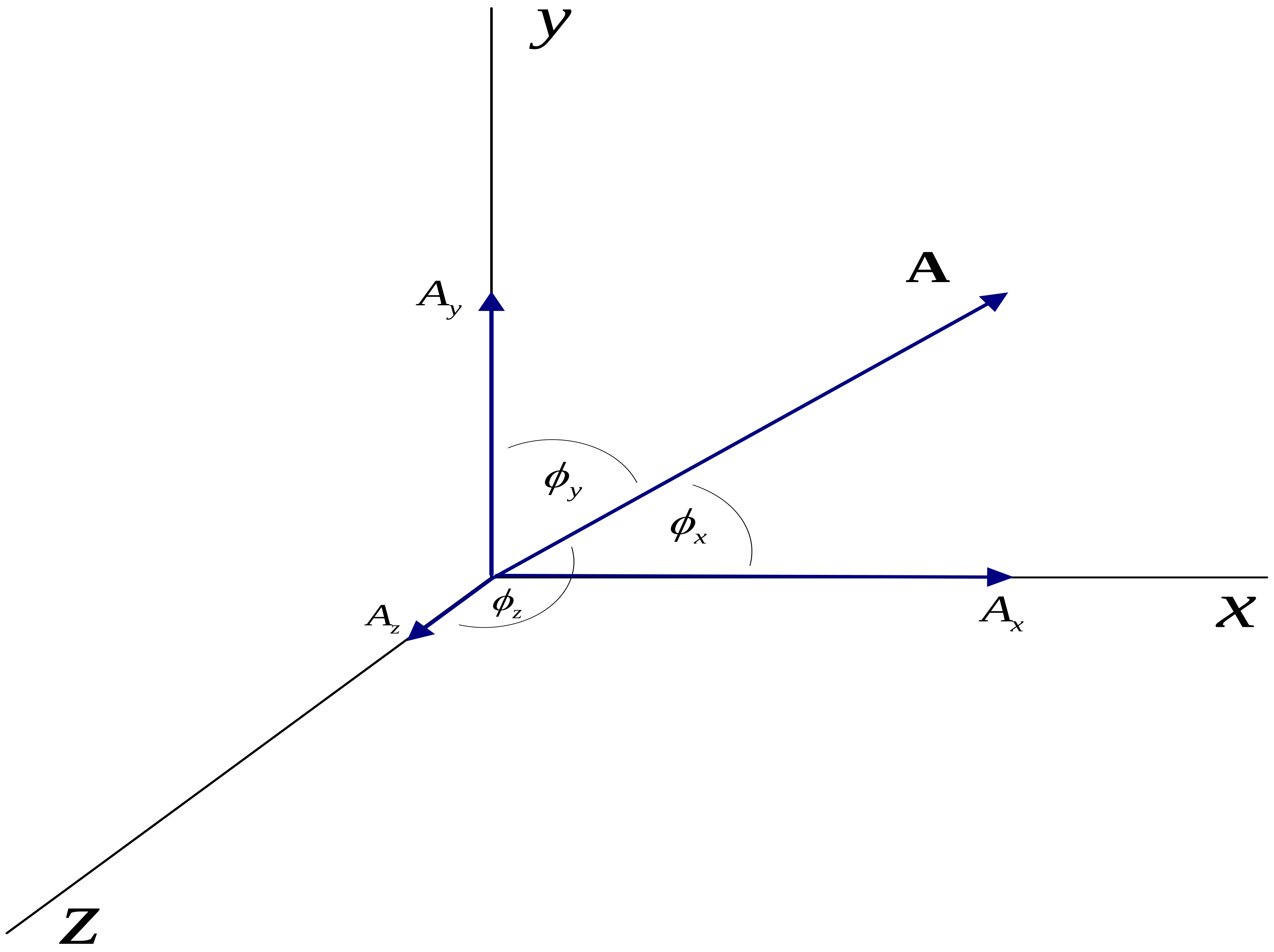
$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

The unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ should not be confused with the imaginary number i or j.

Magnitude or Absolute Value:

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction Angles



Relationships for Direction Angles

$$\cos\phi_x = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\phi_y = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

$$\cos\phi_z = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Example: A force has *x*, *y*, and *z* components of 3, 4, and –12 N, respectively. Express the force as a vector in rectangular coordinates. Determine the magnitude of the force.

$$\mathbf{F} = 3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$$

$$F = \sqrt{(3)^2 + (4)^2 + (-12)^2}$$
$$= 13 \text{ N}$$

Example: Determine the three direction angles for the force of Example.

$$\cos\phi_x = \frac{A_x}{A} = \frac{3}{13} = 0.2308$$

$$\phi_x = \cos^{-1} 0.2308 = 76.66^\circ = 1.338 \text{ rad}$$

$$\cos\phi_y = \frac{A_y}{A} = \frac{4}{13} = 0.3077$$

$$\phi_y = \cos^{-1} 0.3077 = 72.08^\circ = 1.258 \text{ rad}$$

Example: Continuation.

$$\cos\phi_z = \frac{A_z}{A} = \frac{-12}{13} = -0.9231$$

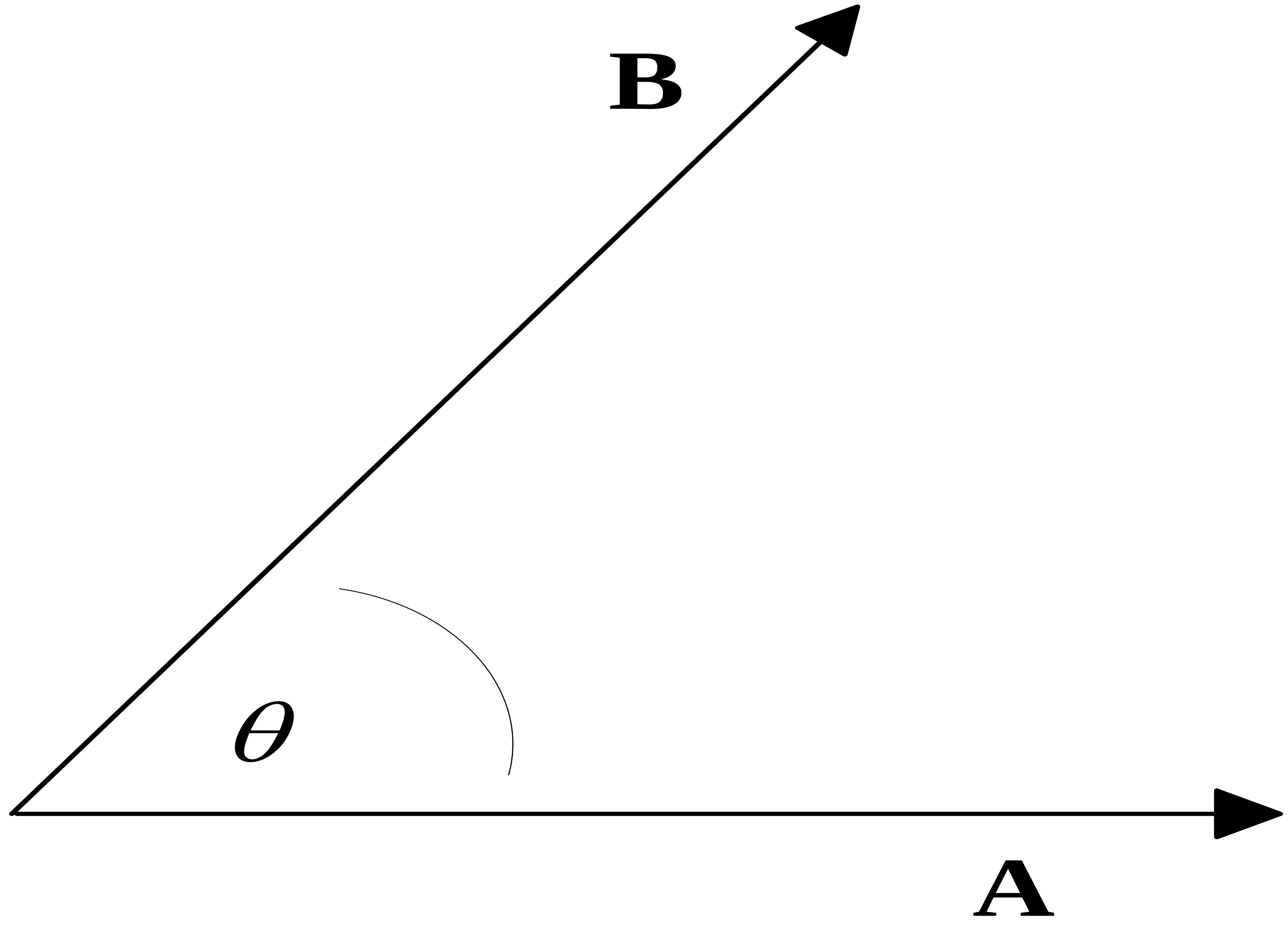
$$\phi_z = \cos^{-1}(-0.9231) = 157.4^\circ = 2.747 \text{ rad}$$

Note that since the z-component is negative, the angle of the vector with respect to the positive z-axis is greater than 90°.

Vector Operations to be Considered

- Scalar or Dot Product **A•B**
- Vector or Cross Product **AxB**
- Triple Scalar Product **(AxB)•C**

Consider two vectors **A** and **B** oriented in different directions.



Scalar or Dot Product

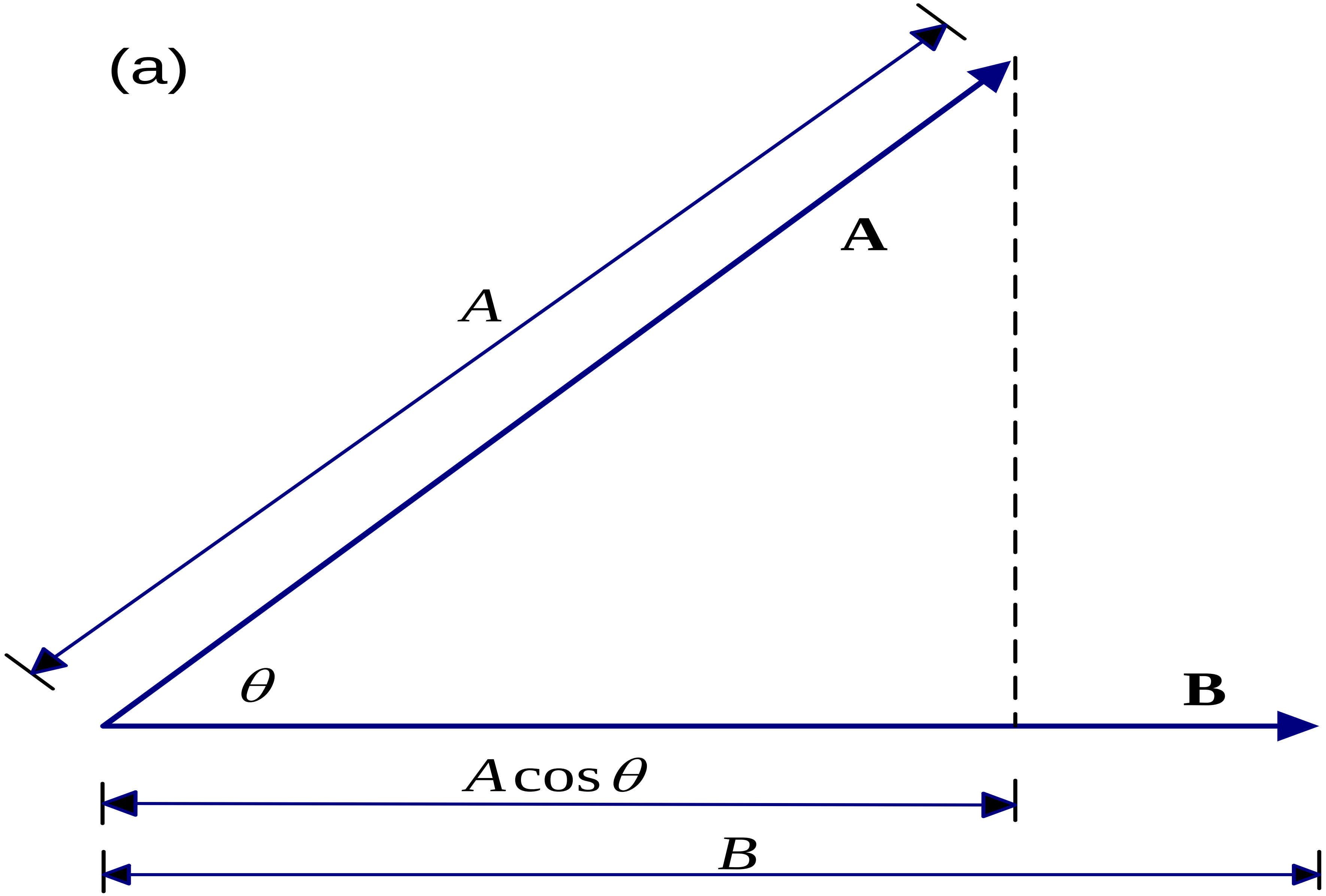
Definition:

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$

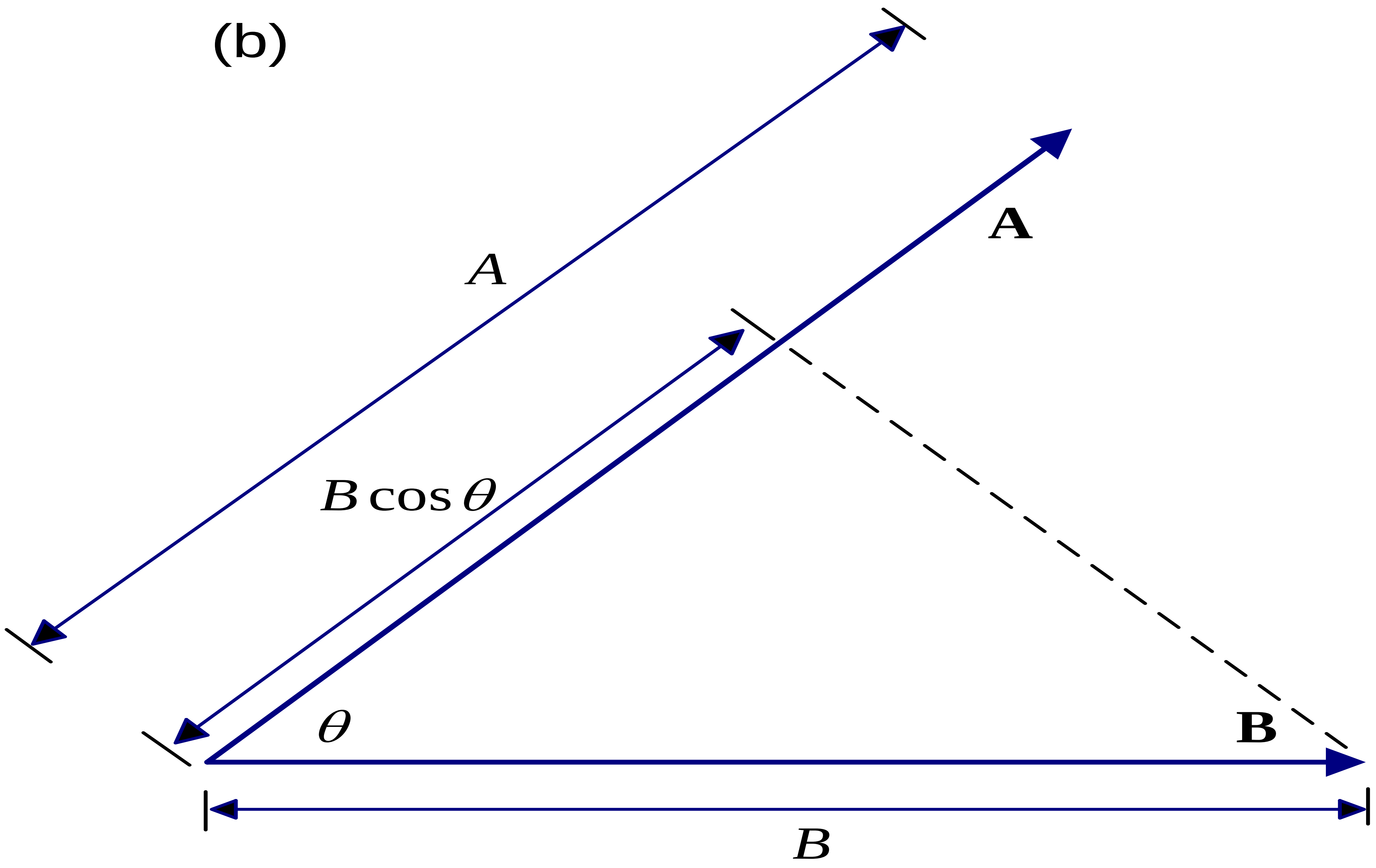
Computation:

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

First Interpretation of Dot Product:
Projection of **A** on **B** times the length of **B**.



Second Interpretation of Dot Product:
Projection of **B** on **A** times the length of **A**.



Some Implications of Dot Product

$$\theta = 0^\circ$$

The vectors are parallel to each other and

$$\mathbf{A} \cdot \mathbf{B} = AB$$

$$\theta = 90^\circ$$

The vectors are $\perp$ to each other and

$$\mathbf{A} \cdot \mathbf{B} = 0$$

Example 14-4. Perform several scalar operations on the following vectors:

$$\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\mathbf{B} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$= \sqrt{(2)^2 + (-2)^2 + (1)^2} = 3$$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$= \sqrt{(3)^2 + (4)^2 + (12)^2} = 13$$

Example 14-4. Continuation.

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= (2)(3) + (\text{-}2)(4) + (1)(12) = 10$$

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta$$

$$\cos\theta = \frac{\mathbf{A} \bullet \mathbf{B}}{AB} = \frac{10}{3 \times 13} = \frac{10}{39} = 0.2564$$

$$\theta = \cos^{-1} 0.2564 = 75.14^{\circ} = 1.311\,\text{rad}$$

Vector or Cross Product

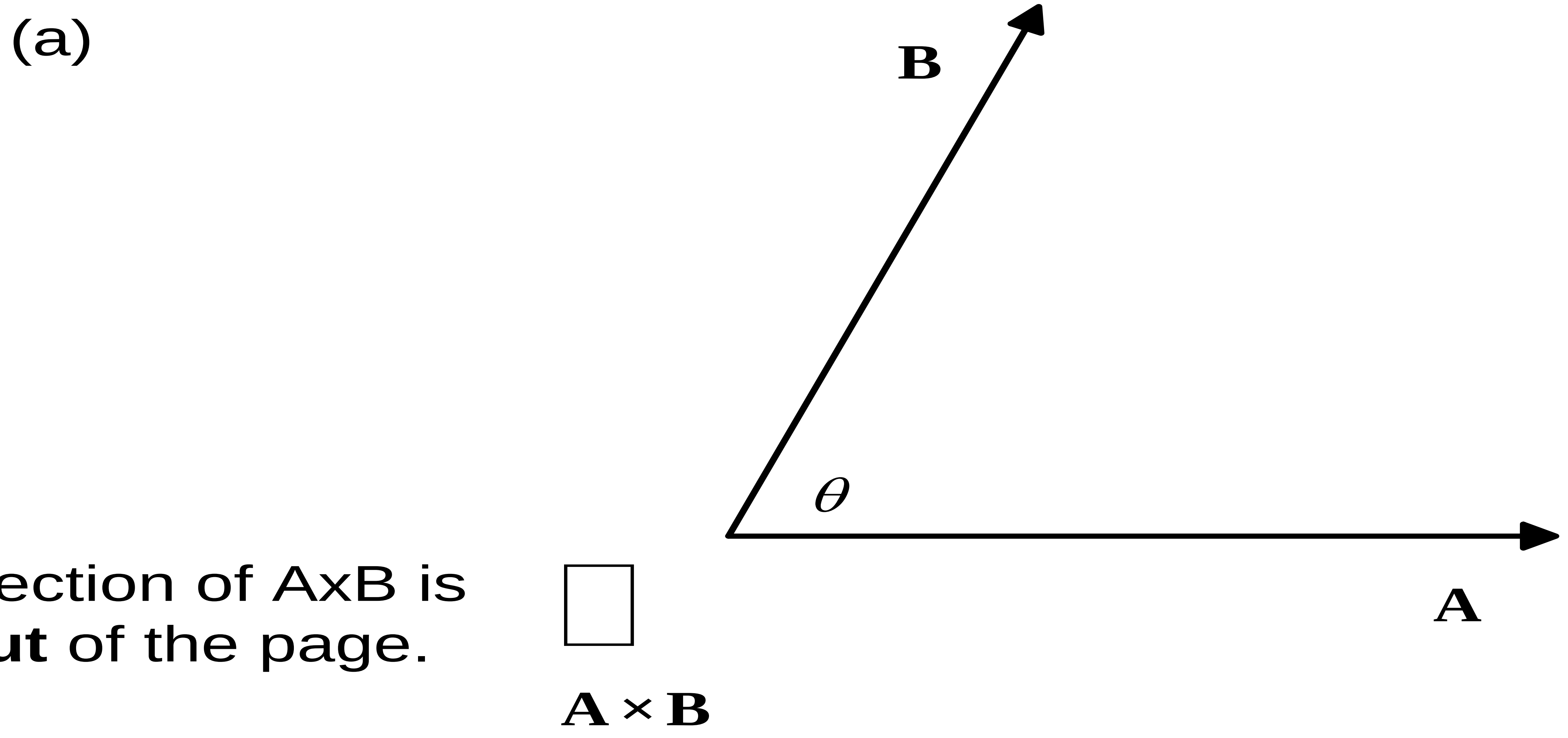
Definition:

$$\mathbf{A} \times \mathbf{B} = \left(AB \sin\theta \right) \mathbf{u_n}$$

Computation:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

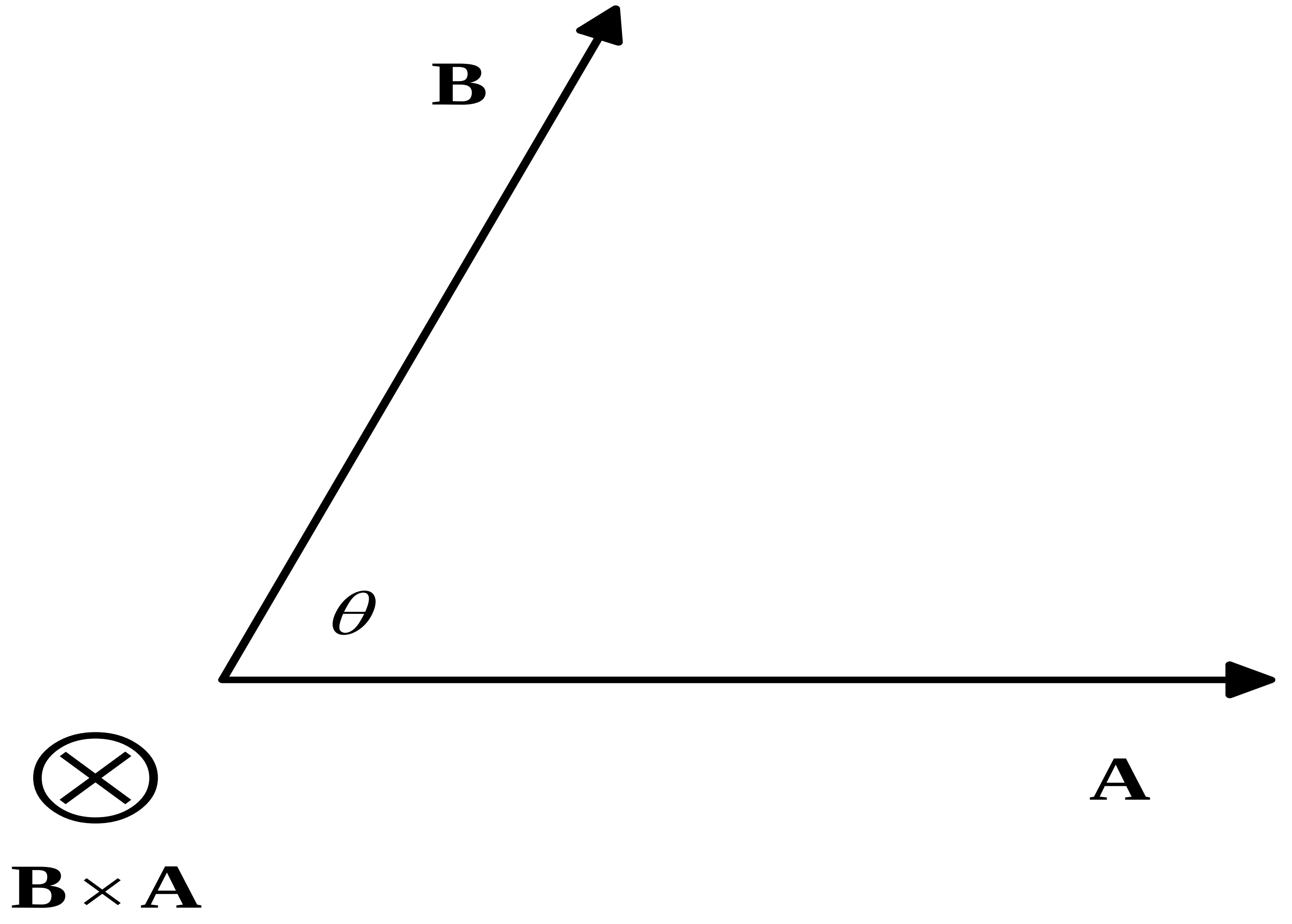
Cross Product AxB



Direction of AxB is **out** of the page.

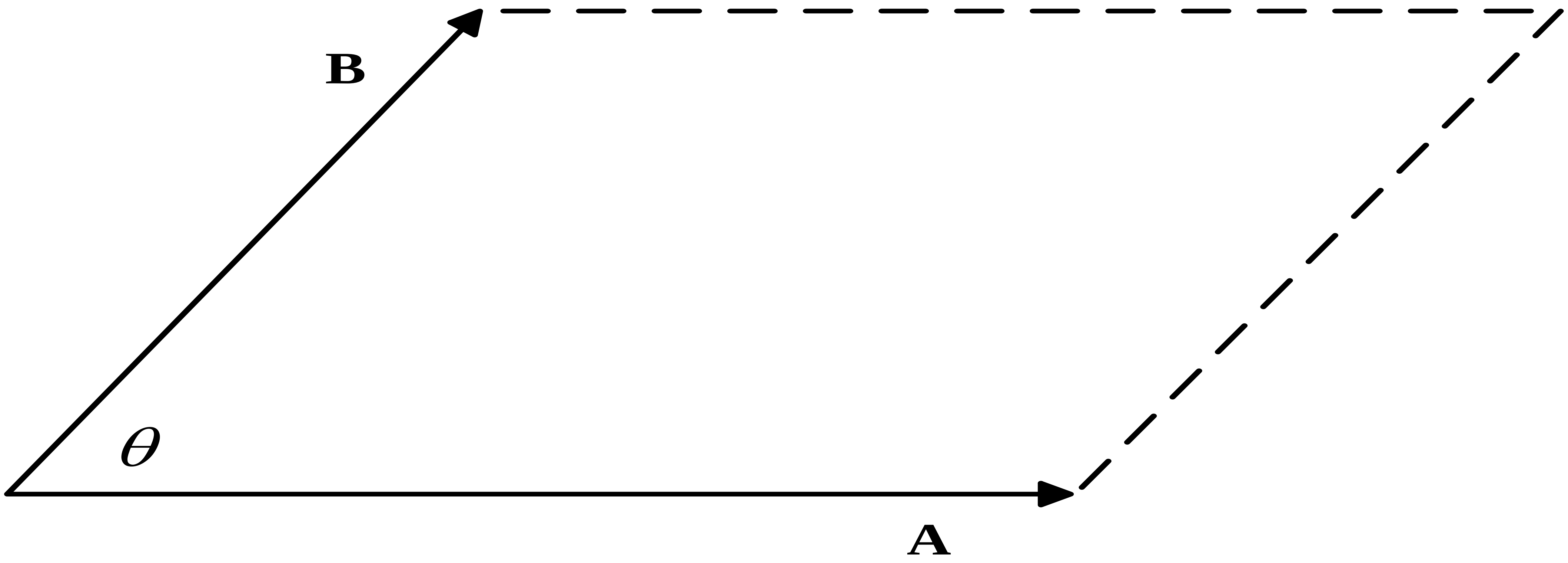
Cross Product BxA

(b)



Direction of BxA is **into** the page.

Area of parallelogram below is the magnitude of the cross product.



Some Implications of Cross Product

$$\theta = 0^\circ$$

The vectors are parallel to each other and

$$\mathbf{A} \times \mathbf{B} = 0$$

$$\theta = 90^\circ$$

The vectors are $\perp$ to each other and

$$\mathbf{A} \times \mathbf{B} = (AB)\mathbf{u_n}$$

Example 14-5. Determine the cross product of the vectors of Example 14-4.

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & -2 & 1\\ 3 & 4 & 12\end{vmatrix}$$

$$\mathbf{A}\times\mathbf{B}=\left[(-2)(12)-(1)(4)\right]\mathbf{i}-\left[(2)(12)-(1)(3)\right]\mathbf{j}$$
$$+\left[(2)(4)-(-2)(3)\right]\mathbf{k}$$
$$=-28\mathbf{i}-21\mathbf{j}+14\mathbf{k}$$

Example 14-6. Determine a unit vector perpendicular to the vectors of Examples 14-4 and 14-5.

$$\left|\mathbf{A}\times\mathbf{B}\right| = \sqrt{(-28)^2 + (-21)^2 + (14)^2} = 37.70$$

$$\mathbf{u_n} = \frac{\mathbf{A}\times\mathbf{B}}{\left|\mathbf{A}\times\mathbf{B}\right|} = \frac{-28\mathbf{i} - 21\mathbf{j} + 14\mathbf{k}}{37.70}$$

$$= -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

Triple Scalar Product

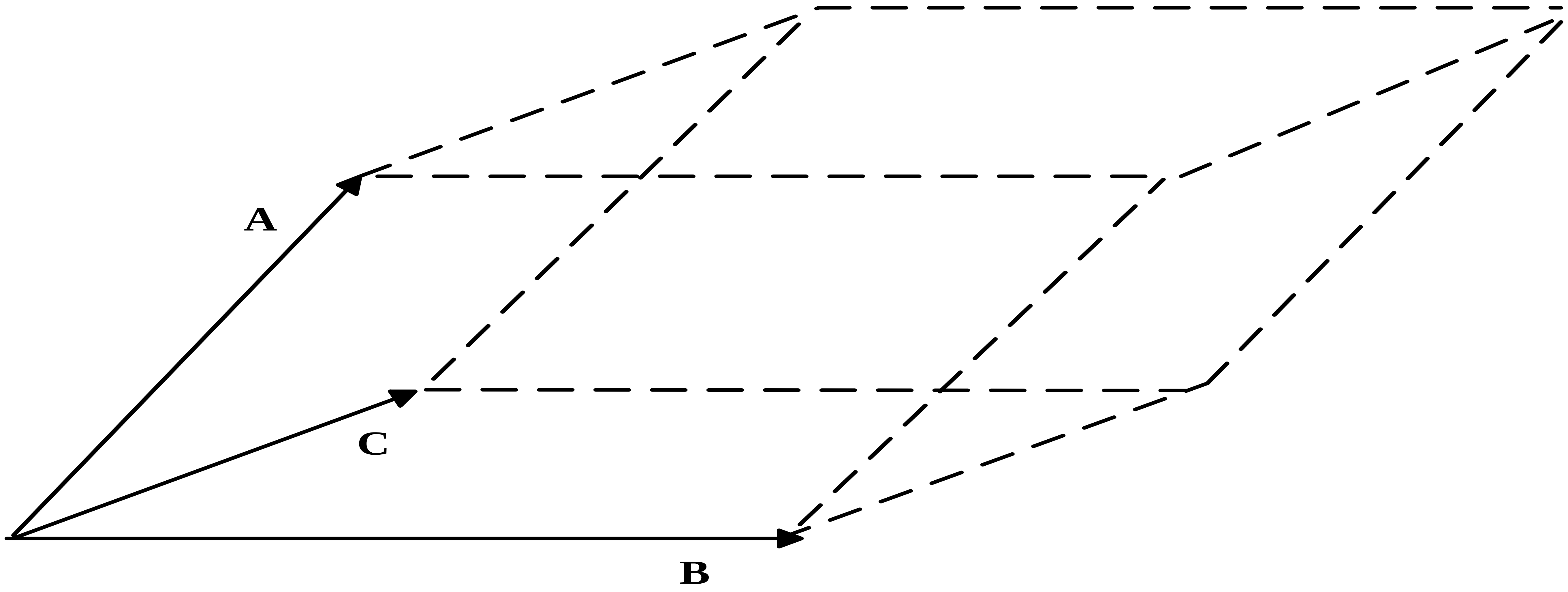
Definition:

$$(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$$

Computation:

$$(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

Volume of parallelepiped below is the triple scalar product of the vectors.



Example 14-7. Determine the triple scalar product of the vectors

$$\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\mathbf{B} = 3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}$$

$$\mathbf{C} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$$

$$(\mathbf{A} \times \mathbf{B}) \bullet \mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} 2 & -2 & 1 \\ 3 & 4 & 12 \\ 3 & 5 & -6 \end{vmatrix}$$

$$= 2(-24 - 60) + 2(-18 - 36) + (15 - 12)$$

$$= -168 - 108 + 3 = -273$$

Work and Energy

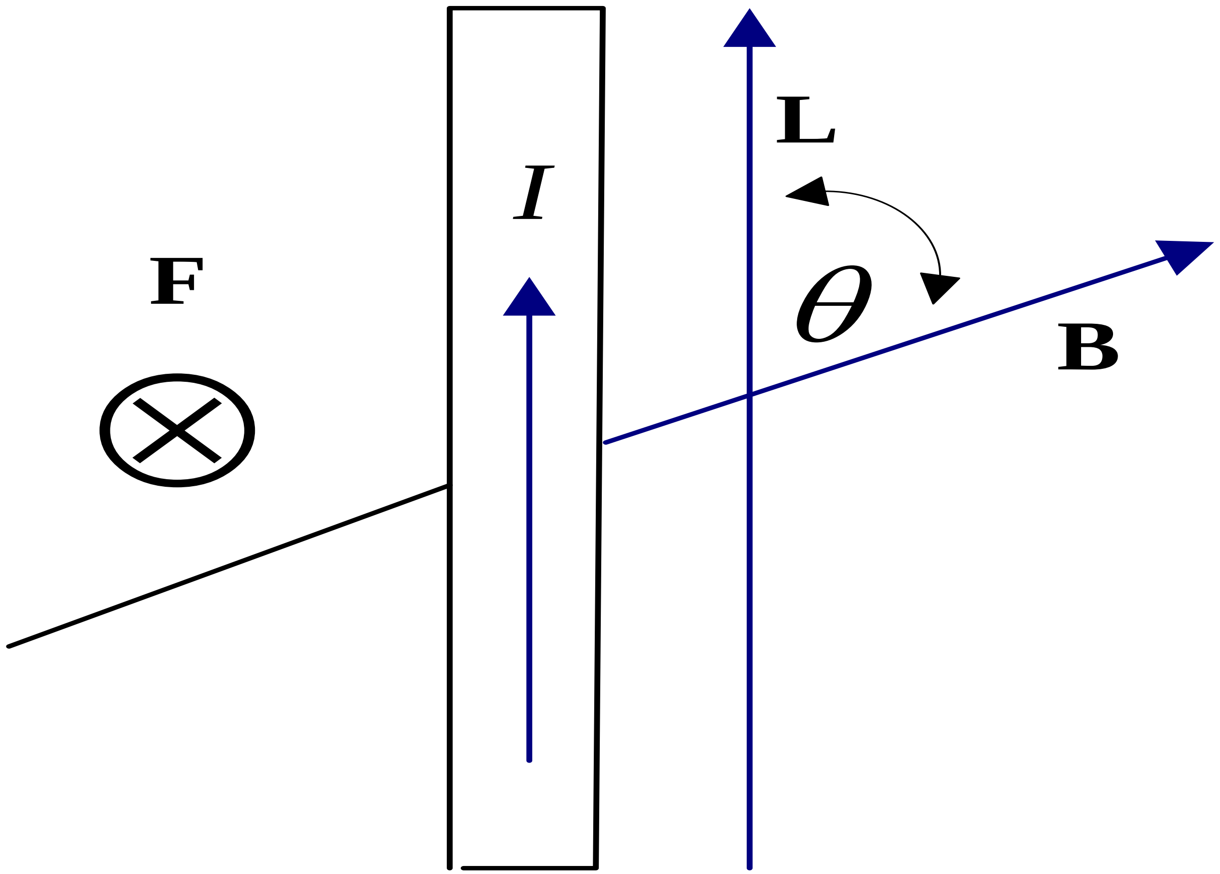
- Let **F** represent a constant force vector and let **L** represent a vector path length over which the work *W* is performed. The first equation below will determine the work. If the force is a function of the position, the differential form is required.

$$W = \mathbf{F} \bullet \mathbf{L}$$

$$dW = \mathbf{F} \bullet \mathbf{dL}$$

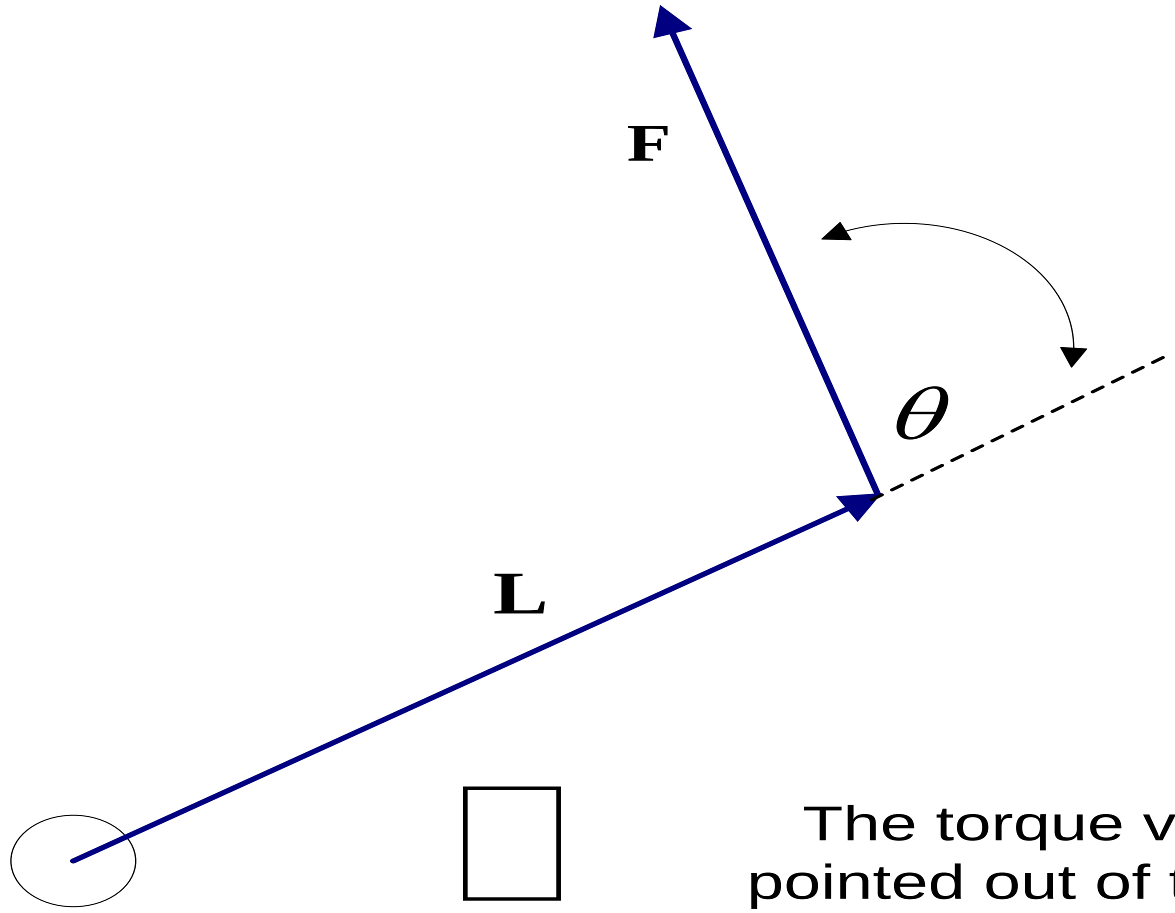
Force on Current Carrying Conductor

The force vector is directed into the page.



$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Force Resulting in Torque Vector



$$\mathbf{T} = \mathbf{L} \times \mathbf{F}$$

Voltage Induced in Moving Conductor

- Assume that a conductor of vector length **L** is moving with vector velocity **v** through a magnetic field vector **B**. The voltage measured across the length is given by the triple scalar product that follows.

$$V = \mathbf{v} \times \mathbf{B} \bullet \mathbf{L}$$

MATLAB Dot Product

- >> A = [Ax Ay Az]

- >> B = [Bx By Bz]

- >> P_dot = dot(A, B)

- The magnitude of a vector A can be determined by the following command:

MATLAB Cross Product

- >> A = [Ax Ay Az]

- >> B = [Bx By Bz]

- >> P_cross = cross(A,B)

MATLAB Triple Scalar Product

- >> A = [Ax Ay Az]
- >> B = [Bx By Bz]
- >> C = [Cx Cy Cz]
-
- >> P_triple = det([A; B; C])